Thermal Runaway Due to Strain-Heating Feedback

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A one-dimensional, dynamic, thermomechanical model, which includes nonlinear inelastic deformation, internal heat generation (strain heating), temperature-dependent inelastic material properties, thermal expansion, and thermoelastic coupling, is considered for a uniform thin bar with one end fixed and insulated and the other end subjected initially to a mechanical load. A nonlinear Maxwell material is examined in this model, and special attention is focused on the temperature changes. By solving a quasistatic nonlinear problem, it is shown that a thermal instability, called thermal runaway, may result because of the strong mutual feedback between strain-heating and the temperature-dependent inelastic material properties. A linearized perturbation study of a bar subjected to a mechanical or a thermal disturbance at the free end then shows that the occurrence of this instability for this related linear problem depends on the choice of the material, the steady-state values of stress, and temperature, and on the characteristic length of the bar, rather than on the magnitude or the form of the disturbance. It is also found in this particular study that thermal expansion, inertia, and thermoelastic coupling have a relatively minor effect on this instability. Aluminum is taken as an example for numerical demonstration in both the linearized and the nonlinear problems.

Introduction

IT is well known from thermodynamics that inelastic deformation may result in internal energy dissipation,[1,2] and the conversion of this dissipative mechanical energy into heat is known as strain heating. Tauchert[3,4] and Dillon[5-7] have studied this phenomenon both analytically and experimentally for various viscoelastic materials under cyclic torsional loadings and significant temperature rises, e.g., 500 K in Ref. 5, were observed. Recently, Allen[8] investigated the strain-heating effect for a viscoplastic rod subjected to cyclic tensional loading and also concluded that strain heating may be an important factor when considering inelastic materials under cyclic loading. In addition to the strain-heating effect, Schapery[9,10] also included the temperature dependence of the material properties in the analysis of linear viscoelastic rods subjected to cyclic shear loading. In these studies, the temperature dependence of the viscoelastic material properties was modeled by a power law, which permitted closed-form integrations in obtaining quasistatic solutions. An important thermal instability, which may be called thermal runaway, was found. Subsequent to these analytical studies, Schapery and Cantey[11] presented experimental results for this thermomechanical system. A similar analysis was also given by Huang and Lee.[12] One of the goals in this paper is to study further the relation between strain-heating and temperature-dependent material properties, but now for noncyclic loading situations and for another viscoelastic material that is nonlinear and contains an Arrhenius-type of dependence on temperature.

It is also known from thermoelasticity theory[1,13] that thermal expansion and thermoelastic coupling may provide another feedback mechanism between mechanical and thermal responses in structural components. As discussed by Dillon and Tauchert,[14] the role of thermoelastic coupling in linear thermoelasticity theory is like damping in an oscillator; the significance of this effect on the thermal runaway phenomenon requires further study. Thus, another goal in this paper is to study a thermomechanical system that includes both of these feedback mechanisms (i.e., strain heating and thermoelastic coupling) for a one-dimensional dynamic model. Then, based on this rather general model, a mathematical study will be given for a nonlinear Maxwell material with temperature-dependent viscosity of the commonly seen Arrhenius type. By using this simple but widely used nonlinear material model, we may extract useful information so as to better understand the relative roles of each effect considered and the possible causes of the thermal runaway instability. To facilitate this process, a uniform thin bar, with one end fixed and insulated and with the other end subjected to a mechanical load in the axial direction, will be studied. As previously noted, thermal expansion and thermoelastic coupling will be included in the material constitutive law and in the energy equation, respectively. Furthermore, the energy equation will also contain the strain-heating term. These two equations, together with the conservation of momentum and strain-displacement relations, form a set of coupled nonlinear partial differential equations. Particular emphasis will be placed on the evaluation of the temperature changes. We also point out that shear band formation[15] and necking[16] effects, which are usually associated with strain-rate sensitive materials, may also be important if thermal softening is present. However, these effects will not be considered in this paper.

This paper is divided into two parts. A nonlinear problem is solved first, with the assumption that the mechanical load is applied so slowly that the inertia effect can be ignored. The resulting problem is decoupled mathematically, and thus the temperature field can be determined by an iterative procedure with the use of a Green's function. The solution to this nonlinear quasistatic problem clearly demonstrates the thermal runaway phenomenon. Next, employing a perturbation technique for a related problem, linearized equations that are amenable to a stability analysis are obtained and studied. Solutions for dynamic conditions are obtained by the method of separation of variables, and a stability analysis is performed with the use of the Routh-Hurwitz criterion. Analytical solutions under quasistatic conditions are also obtained both for the case of a mechanical disturbance and for a thermal disturbance. Numerical results are presented for an aluminum bar in both the linearized and the nonlinear problems.

The thermal runaway phenomenon is found to occur for a critical condition, as the temperature rise due to strain heating

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results in a reduction of the inelastic material properties (e.g., the viscosity coefficient in the case of nonlinear Maxwell materials), and the material softens. Without an appropriate external dissipating agency, material softening may in turn generate greater inelastic strain and the associated strain heating. In this particular study, it is found that thermal expansion, thermoelastic coupling, and inertia have a relatively minor effect on the occurrence of this instability. It should be noted that thermal dissipating mechanisms, such as convection, are often present in conventional engineering applications such that thermal runaway may not be a serious problem. However, in the absence of such dissipating mechanisms, or if thermal dissipative mechanisms are very weak (e.g., as in space), thermal runaway may become important.

Nonlinear Problem

Consider a thin bar of length L with one end ($x = 0$) fixed and with the other end ($x = L$) subject to an axial stress load $p(t)$; the bar is initially at a uniform temperature T_0. The constitutive law for a very wide class of inelastic materials may be expressed as

$$\frac{\partial \epsilon}{\partial t} = \frac{1}{E}\frac{\partial \sigma}{\partial t} + \alpha \frac{\partial T}{\partial t} + f(\sigma,\epsilon;T) \tag{1}$$

where $\epsilon(x,t)$, $\sigma(x,t)$, and $T(x,t)$ are, respectively, the normal strain, normal stress, and temperature. These quantities in general vary with the axial coordinate x and the time t. The first term and the second term on the right-hand side of Eq. (1) represent, respectively, the instantaneous linear elastic response and the thermal expansion, which are characterized by Young's modulus E and by the linear thermal expansion coefficient α. The last term in Eq. (1) is the inelastic strain rate, which depends directly on σ and ϵ with the dependence on T occurring indirectly through the temperature dependence of the material properties. Various forms for f have been postulated for different materials and for different loading situations (e.g., see Ref. 17).

The conservation of energy equation consistent with Eq. (1) (e.g., see Chang and Cozzarelli[18] for nonlinear thermoviscoelastic materials) is given as

$$k\frac{\partial^2 T}{\partial x^2} + \sigma f(\sigma,\epsilon;T) - \alpha T_0 \frac{\partial \sigma}{\partial t} = \rho c \frac{\partial T}{\partial t} \tag{2}$$

where k, ρ, and c are the thermal conductivity, mass density, and heat capacity of the material, respectively. The second term in Eq. (2) is known as the strain-heating term, whereas the third term represents the thermoelastic coupling effect. Note that heat flux through the lateral surface has been neglected, i.e., the lateral surface of the bar is assumed to be insulated. Equations (1) and (2), together with the conservation of momentum equation

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \tag{3}$$

and the strain-displacement relation

$$\epsilon = \frac{\partial u}{\partial x} \tag{4}$$

where $u(x,t)$ is the axial displacement, form a set of equations that completely describes the response of the system. It is implied in Eq. (3) that the cross-sectional area of the bar is constant during the deformation process, and thus the necking phenomenon, which may make a significant contribution to structural instability, is ignored. Body forces are also assumed to be negligible.

In particular, a nonlinear Maxwell model, which employs for the inelastic strain rate a Norton-type power law with temperature-dependent viscosity, will be considered here. For this type of material, Eqs. (1–3) can be rewritten with the use of Eq. (4) as

$$\frac{\partial v}{\partial x} = \frac{1}{E}\frac{\partial \sigma}{\partial t} + A(T)\sigma^n + \alpha \frac{\partial T}{\partial t} \tag{5}$$

where

$$A(T) = A_0 e^{-B/T} \tag{6}$$

is the reciprocal viscosity of the widely seen Arrhenius type, and

$$k\frac{\partial^2 T}{\partial x^2} + A(T)\sigma^{n+1} - \alpha T_0 \frac{\partial \sigma}{\partial t} = \rho c \frac{\partial T}{\partial t} \tag{7}$$

$$\frac{\partial \sigma}{\partial x} = \rho \frac{\partial v}{\partial t} \tag{8}$$

The variable $v(x,t)$ is the axial velocity, and A_0, B, and n are, respectively, the pre-exponential constant in the reciprocal viscosity, the creep activation constant, and the stress power. Equations (5), (7), and (8) represent a system of coupled, nonlinear partial differential equations for a nonlinear Maxwell material in terms of the dependent variables v, σ, and T. In general, it is difficult to solve these equations because of the presence of strong nonlinearities and coupling.

Now, let us consider a special case in which the characteristic time t_0 of the applied stress is large enough so that inertia effects can be neglected. It follows immediately from this assumption that the stress field is uniformly distributed and simply equal to the applied stress at the free end. As a result, the problem is uncoupled, and thus the temperature and velocity fields can be obtained successively. As an example, it is assumed that the bar is insulated at the fixed end and maintained at a constant temperature T_0 at the free end. The Green's function associated with the auxiliary problem, in which the known heat source terms in Eq. (7) (i.e., the second and the third terms) are suppressed, is simply

$$G(x,t\,|\,x',\tau) = \frac{2}{L}\sum_{m=1}^{\infty} \exp\left[-\frac{k}{\rho c}\beta_m^2 (t-\tau)\right] \times \cos\beta_m x \cos\beta_m x' \tag{9}$$

where $\beta_m = (2m-1)\pi/2L$. If an initial guess for the solution is available, say $T^{(0)}(x,t)$, then the following integrals (e.g., see Ref. 19)

$$T^{(i+1)}(x,t) = \frac{1}{\rho c}\int_{\tau=0}^{\tau=t}\int_{x'=0}^{x'=L} G(x,t\,|\,x',\tau) \times \left\{A[T^{(i)}(x',\tau)]p(\tau)^{n+1} - \alpha T_0 \frac{\partial p(\tau)}{\partial \tau}\right\} \mathrm{d}x'\,\mathrm{d}\tau, \quad i = 0,1,2,\ldots \tag{10}$$

can be carried out sequentially by numerical methods. The rate of convergence of this iterative process depends greatly on the initial guess. Usually, the solution for the completely insulated case (which can be easily determined) will be a reasonable choice for the initial guess.

The solution for an aluminum bar subjected to an applied stress of the exponential form

$$p(t) = \sigma_0(1 - e^{-t/t_0}) \tag{11}$$

at the free end is shown in Figs. 1 and 2. The material properties and other necessary parameters are given in Table 1; the nonlinear Maxwell material data were taken from Garafalo[20] and Walter and Ponter.[21] Figure 1 shows the temperature increase with time at $x = 0$, $L/2$ and $3L/4$, and Fig. 2 shows the temperature distribution over x at $t = 250$, 500, 750, and 1000 s. These two figures clearly indicate that without appropriate heat dissipating mechanisms the temperature can increase without bound. It is found in this particular example that thermal expansion and thermoelastic coupling have a relatively minor effect on the temperature rise. However, we must also point out that the role of inertia and variable cross section in the thermal runaway phenomenon requires further study for this full nonlinear problem.

Linear Problem—Stability Analysis

It was demonstrated in the previous section that the temperature in the rod can increase significantly as a result of strain heating. Excessive increases in temperature are often associated with material failure. Therefore, a stability analysis is of interest so that conditions associated with unstable behavior can be identified. The means chosen for this analysis involve a perturbation from an initial steady-state condition. If an initial steady state is to exist, there must be some external cooling device that in the steady state extracts the heat generated by strain heating. It is clear from Eq. (7) that a cooling rate of $S_0 = -A(T_0)\sigma_0^{n+1}$ is required for such a device to maintain the bar at reference temperature T_0 for constant stress σ_0. Under

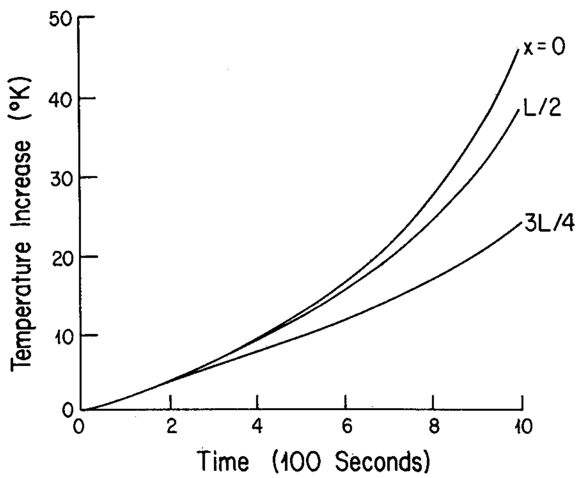

Fig. 1 Nonlinear problem—temperature increase with t at $x = 0$, $L/2$, $3L/4$.

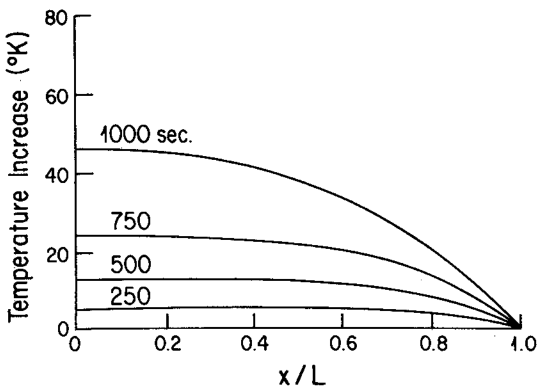

Fig. 2 Nonlinear problem—temperature variation with x for t = 250, 500, 750, 1000 s.

Table 1 Material properties for aluminum and other parameters

ρ = 2707 kg/m^3	B = 13603 K
c = 986 J/kg K	L = 1 m
k = 222 W/m K	$\sigma_0 = 3.5 \times 10^7$ Pa
$E = 5.52 \times 10^{10}$ Pa	T_0 = 533 K
n = 4.55	t_0 = 10 s
$A_0 = 7.836 \times 10^{-27}$ Pa$^{-4.55}$/s	$\sigma_1/\sigma_0 = 0.05$
$\alpha = 2.3 \times 10^{-5}$ 1/K	$T_1/T_0 = 0.01$

this condition, the steady-state velocity is obtained from Eq. (5) as

$$v_0 = \hat{v}_0 x/L$$

where

$$\hat{v}_0 = A(T_0)\sigma_0^n L$$

If the system then is subjected to a small additional external disturbance in stress or temperature at $x = L$, this disturbance will result in additional increments of stress $\tilde{\sigma}$, temperature $\tilde{T}$, and velocity $\tilde{v}$. It is assumed that the magnitudes in the steady state are much larger than the magnitudes of the increments in the perturbed state, and we may then write

$$T = T_0 + \tilde{T}, \qquad T_0 >> \tilde{T} \tag{12a}$$

$$\sigma = \sigma_0 + \tilde{\sigma}, \qquad \sigma_0 >> \tilde{\sigma} \tag{12b}$$

$$v = v_0 + \tilde{v}, \qquad v_0 >> \tilde{v} \tag{12c}$$

Substituting Eq. (12) into Eqs. (5-8), subtracting the relation for the steady state [with cooling term S_0 added to Eq. (7)], retaining the linear terms in the increments (as in Cozzarelli et al.[22]), and then introducing the nondimensional quantities

$$\bar{T} = \frac{\tilde{T}}{T_0},\ \bar{\sigma} = \frac{\tilde{\sigma}}{\sigma_0},\ \bar{v} = \frac{\tilde{v}}{\hat{v}_0},\ \bar{x} = \frac{x}{L},$$

$$\bar{t} = \frac{t}{L/\hat{v}_0},\ \bar{E} = \frac{E}{\sigma_0},\ \bar{\rho} = \frac{\rho}{\sigma_0/\hat{v}_0^2},\ \bar{\alpha} = \frac{\alpha}{1/T_0},$$

$$\bar{k} = \frac{k}{L\hat{v}_0\sigma_0/T_0},\ \bar{c} = \frac{c}{\hat{v}_0^2/T_0},\ \eta = \frac{B}{T_0},\ \bar{\sigma}_1 = \frac{\sigma_1}{\sigma_0},\ \bar{T}_1 = \frac{T_1}{T_0}$$

we obtain the following coupled linear equations:

$$\frac{\partial v}{\partial x} = \frac{1}{E}\frac{\partial \sigma}{\partial t} + n\sigma + \eta T + \alpha\frac{\partial T}{\partial t} \tag{13}$$

$$\frac{\partial \sigma}{\partial x} = \rho\frac{\partial v}{\partial t} \tag{14}$$

$$k\frac{\partial^2 T}{\partial x^2} + (n+1)\sigma + \eta T - \alpha\frac{\partial \sigma}{\partial t} = \rho c\frac{\partial T}{\partial t} \tag{15}$$

Note that the overbars have been deleted in Eqs. (13-15) for the sake of convenience of notation. The associated boundary and initial conditions are

$$v = 0, \frac{\partial T}{\partial x} = 0 \qquad \text{at } x = 0 \tag{16a}$$

$$\sigma = \sigma_1, T = T_1 \qquad \text{at } x = 1 \tag{16b}$$

$$T = \sigma = v = 0 \qquad \text{at } t = 0 \tag{16c}$$

where $\sigma_1 = 0$ for a purely thermal disturbance, and $T_1 = 0$ for a purely mechanical disturbance.

It can be seen that for the boundary conditions given in Eq. (16), the method of separation of variables may be employed if we divide Eqs. (13-16) into two problems (see Ref. 23), i.e., a steady-state problem with inhomogeneous boundary conditions and a transient problem with homogeneous boundary conditions. Solving these two problems and superposing solutions, we obtain

$$T = T_i + \sum_{m=1}^{\infty} p_m(t)\cos\beta_m x \tag{17}$$

$$\sigma = \sigma_i + \sum_{m=1}^{\infty} q_m(t)\cos\beta_m x \tag{18}$$

$$v = v_i + \sum_{m=1}^{\infty} r_m(t) \sin\beta_m x \tag{19}$$

where

$$\beta_m = (2m - 1)\pi/2$$

The temporal functions $p_m(t)$, $q_m(t)$, $r_m(t)$ are yet to be determined. In Eqs. (17–19), T_i, σ_i, and v_i are the following solutions for the steady-state problem:

$$T_i = \frac{n+1}{\eta} [(\cos\xi x/\cos\xi) - 1]\, \sigma_1$$

$$\sigma_i = \sigma_1$$

$$v_i = \sigma_1[-x + (n+1)(\cos\xi x/\xi \cos\xi)]$$

for a purely mechanical disturbance, and

$$T_i = T_1 (\cos\xi x/\cos\xi)$$

$$\sigma_i = 0$$

$$v_i = T_1\eta (\sin\xi x/\xi \cos\xi)$$

for a purely thermal disturbance, where

$$\xi = (\eta/k)^{1/2}$$

Substituting Eqs. (17–19) into Eqs. (13–15), multiplying by $\cos\beta_n x$, $\sin\beta_n x$, and $\cos\beta_n x$ ($n = 1,2,\ldots$), respectively, integrating the resulting equations over the space domain ($x = 0$ to $x = 1$) and invoking the orthogonality properties, the quantities $p_m(t)$, $q_m(t)$, and $r_m(t)$ in Eqs. (17–19) can be solved approximately for $m = 1,2,\ldots,M$. Accordingly, we obtain the formulation

$$Z = A\, Z \tag{20}$$

where

$$Z = [p_1 p_2 \ldots p_M \,|\, q_1 q_2 \ldots q_M \,|\, r_1 r_2 \ldots r_M]^T \qquad (3M \times 1)$$

and

$$A = \begin{bmatrix} \rho c I & \alpha I & 0 \\ \alpha I & 1/EI & 0 \\ 0 & 0 & \rho I \end{bmatrix}^{-1} \times \begin{bmatrix} C & (n+1)I & 0 \\ -\eta I & nI & D \\ 0 & -D & 0 \end{bmatrix} \qquad (3M \times 3M)$$

where I is the identity matrix, and C and D are the diagonal matrices

$$C = \mathrm{diag}(\eta - k\beta_m^2 2), \qquad m = 1,2,\ldots,M$$

$$D = \mathrm{diag}(\beta_m), \qquad m = 1,2,\ldots,M$$

As initial conditions, we have

$$p_m(0) = (-1)^{m+1}[(n + 1/\eta)]2\beta_m\{[1/(\xi^2 - \beta_m^2)] + (1/\beta_m^2)\}\sigma_1$$

$$q_m(0) = (-1)^{m+1}(2/\beta_m)\sigma_1$$

$$r_m(0) = (-1)^{m+1}2\{(1/\beta_m^2) + [(n+1)/(\xi^2 - \beta_m^2)]\}\sigma_1$$

for the purely mechanical disturbance, and

$$p_m(0) = (-1)^{m+1}2\beta_m^2[1/(\xi^2 - \beta_m^2)]T_1$$

$$q_m(0) = 0$$

$$r_m(0) = (-1)^{m+1}[2\eta/(\xi^2 - \beta_m^2)]T_1$$

for the purely thermal disturbance. To obtain accurate results, a reasonable number of terms in Eqs. (17–19) should be considered. However, this will introduce difficulties in the numerical integrations because matrix A is of relatively large order. Also, because of the inertia effects, the matrix A is stiff, and thus a very small time step has to be taken for each integration. Alternatively, transform methods[24] may be employed to solve Eq. (20).

If, as in the nonlinear problem, we again neglect the inertia effects, the solution of Eqs. (13–15) may be found with the use of variable transformations[19] as

$$T(x,t) = \frac{2(n+1)}{\rho c}\sigma_1 \sum_{m=1}^{\infty} \frac{(-1)^m}{\beta_m \lambda_m} (1 - e^{\lambda_m t}) \cos\beta_m x \tag{21}$$

for the purely mechanical disturbance, and

$$T(x,t) = \frac{\cos\xi x}{\cos\xi} T_1 + \sum_{m=1}^{\infty} (-1)^{m+1} 2\beta_m T_1 \frac{1}{\xi^2 - \beta_m^2} e^{\lambda_m t} \cos\beta_m x \tag{22}$$

for the purely thermal disturbance. In Eqs. (21) and (22)

$$\beta_m = (2m - 1)\pi/2$$

$$\lambda_m = (1/\rho c)(\eta - k\beta_m^2)$$

It is interesting to note that the same λ_m appears in Eqs. (21) and (22). This indicates that the perturbations in temperature will follow the same general pattern, regardless of the forms and the magnitudes of the disturbances. Furthermore, for this quasistatic solution to be stable, it is required that

$$\xi^2 = (\eta/k) < \beta_1^2 \tag{23}$$

or in dimensional form

$$\frac{BL^2 A_0 e^{-B/T_0} \sigma_0^{n+1}}{T_0^2 k} < \frac{\pi^2}{4} \tag{24}$$

Using the values of the parameters listed for an aluminum bar in Table 1, it is found that conditions (23) or (24) are violated. Thus, the quasistatic system with these values for the parameters is unstable.

Let us now return to a consideration for stability for the dynamic problem. Substituting Eqs. (17–19) into Eqs. (13–15) and combining the resulting equations into one equation in terms of p_m gives

$$a_1 p_m''' + a_2 p_m'' + a_3 p_m' + a_4 p_m = 0 \tag{25}$$

where

$$a_1 = (1/E)(\rho c - \alpha^2 E) \tag{26a}$$

$$a_2 = (k/E)\beta_m^2 - (\eta/E) + \rho c E + \alpha(n+1) - \alpha\eta \tag{26b}$$

$$a_3 = k\beta_m^2 n + c\beta_m^2 + \eta \tag{26c}$$

$$a_4 = (1/\rho)(k\beta_m^2 - \eta)\beta_m \tag{26d}$$

The primes represent derivatives with respect to the time. By applying the Routh-Hurwitz criterion,[25] one root (for $m = 1$) with a positive real part is found for aluminum using the parameters listed in Table 1. This observation indicates that inertia, thermal expansion, and thermoelastic coupling, which were not considered in obtaining Eqs. (21) and (22), do not in this particular problem contribute to the instability found earlier. Thus, it may be concluded that in this case the mechanisms that may cause thermal instability (runaway) are the temperature dependence in the inelastic material properties and the strain heating. This conclusion can also be obtained by letting $\eta = 0$ (or $B = 0$ in dimensional form) in Eqs. (26). The resulting coefficients in Eq. (25) are then all positive (see Ref. 18

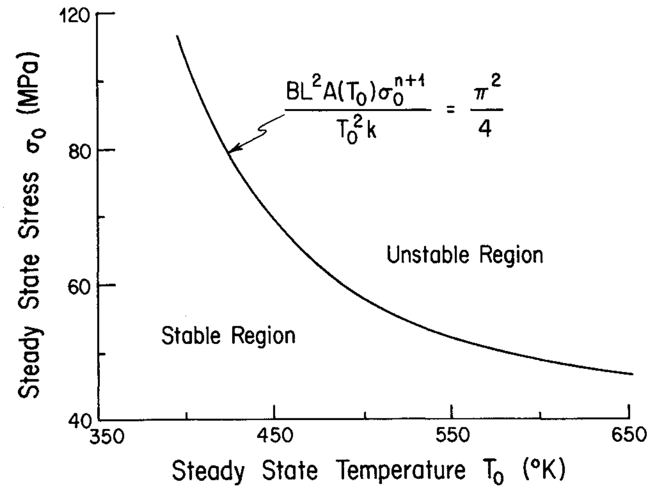

Fig. 3 Linear problem—stability diagram of steady-state stress vs steady-state temperature for an aluminum bar 1 m long.

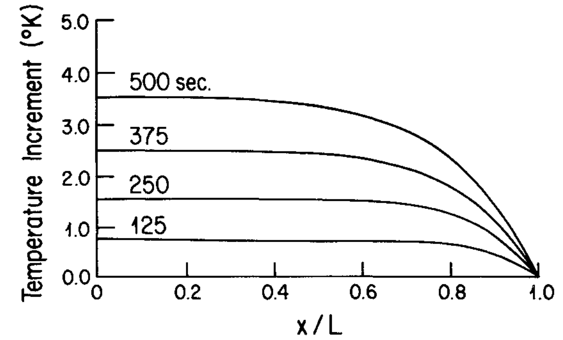

Fig. 4 Linear problem—temperature increment due to mechanical disturbance.

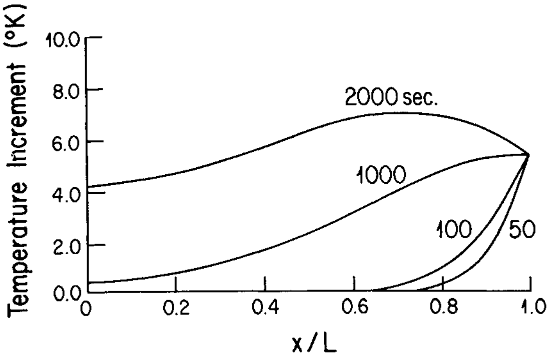

Fig. 5 Linear problem—temperature increment due to thermal disturbance.

for $a_1 > 0$) and

$$a_2a_3 - a_1a_4 = [(kn/E) + (\alpha^2/\rho)]k\beta_m^2$$
$$+ [\rho cn^2k + \rho c^2 + \alpha(n+1)kn + \alpha(n+1)c]\beta_m^2 > 0$$

Using the Routh-Hurwitz criterion again, the system is seen to be stable in this case. Thus, if the temperature dependence of the material properties is not considered, the thermal runaway phenomenon will not be observed.

It is important to note that in accordance with condition (23), the occurrence of thermal instability depends on one nondimensional parameter ξ, whose value is determined by the material properties A_0, B, n, k, the steady-state values T_0, σ_0, and the length of the bar L. Thus, in the linearized problem, the occurrence of runaway is not affected by the magnitude or form of the disturbance. The stability relationship between T_0 and σ_0 [i.e., condition (24)] for an aluminum bar of length 1 m is shown in Fig. 3. Also shown in Figs. 4 and 5 are the temperature increments due to mechanical and thermal disturbances, respectively, where σ_1/σ_0 and T_1/T_0 are given at the end of Table 1.

Concluding Remarks

A one-dimensional, dynamic, thermomechanical model, which includes nonlinear viscoelastic deformation, internal heat generation (strain heating), thermal expansion, thermoelastic coupling, and temperature-dependent material properties, was considered for investigating the possible causes of the thermal runaway phenomenon. A detailed study was given for a special but widely used nonlinear material model (i.e., nonlinear Maxwell), which approximately characterizes some polymers and metallic materials at elevated temperatures. For a quasistatic nonlinear problem under noncyclic loading conditions, significant temperature increases were found for an aluminum material by means of an iterative solution. Further investigation by a perturbation technique showed that the occurrence of thermal instability (i.e., runaway) in a related linear problem depends on the choice of material, steady-state values of stress and temperature, and the length of the bar, but not on the magnitude or the form of the disturbance.

The phenomenon of thermal runaway was clearly demonstrated to be the result of mutual feedback between strain-heating and the temperature-dependent inelastic material properties. If either effect is not included in an analysis, the possibility of thermal runaway will not be explored, and an unexpected failure may occur. The effect on the thermal runaway due to inertia, thermal expansion, and thermoelastic coupling has been shown to be relatively minor for the particular one-dimensional problem considered.

Acknowledgments

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